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List of the main results in the paper

A SUPPLEMENT TO A BINOMIAL INVERSION FORMULA

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Let B_n and $S(n,k)$ be the Bernoulli numbers and the second kind Stirling numbers respectively.

Theorem 1. *We have the following inversion formula:*

$$F(n)=\sum_{k=0}^{n}\binom{n}{k}f(k)-f(n)\ (n=1,2,3,\dots)$$
$$\iff f(n-1)=\frac{1}{n}\sum_{k=1}^{n}\binom{n}{k}F(k)B_{n-k}\ (n=1,2,3,\dots).$$

Theorem 1'. *For any function f and positive integer n we have*

$$\sum_{k=0}^{n}\binom{n}{k}\Big(\sum_{r=0}^{k}\binom{k}{r}f(r)-f(k)\Big)B_{n-k}=nf(n-1).$$

Corollary 3. *We have the following inversion formula:*

$$F(n)=\sum_{k=0}^{n}\binom{n}{k}f(k)+f(n)\ (n=0,1,2,\dots)$$
$$\iff f(n-1)=\frac{1}{n}\sum_{k=0}^{n}\binom{n}{k}F(k)(1-2^{n-k})B_{n-k}\ (n=1,2,3,\dots).$$

Theorem 2'. *For $\lambda\neq 1$ we have*

$$F(n)=\sum_{k=0}^{n}\binom{n}{k}f(k)-\lambda f(n)\ (n=0,1,2,\dots)$$
$$\iff f(n)=-\sum_{m=0}^{n}\binom{n}{m}F(m)\sum_{k=0}^{n-m}\frac{k!S(n-m,k)}{(\lambda-1)^{k+1}}\ (n=0,1,2,\dots).$$